Math 10A
Worksheet, Discussion #27; Tuesday, 7/24/2018
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1 Second-Order Linear ODEs

1.1 Concepts

1. This is for **linear, homogeneous, constant coefficients, second-order** differential equations. It is of the form

$$\boxed{ay'' + by' + cy = 0.}$$

To solve this, we find the roots of the **characteristic polynomial**: $ar^2 + br + c = 0$. Then depending on the roots, we can determine the **general solution**:

Roots	$r_1 \neq r_2$	r, r	$a \pm bi$
General Solution	$c_1e^{r_1t} + c_2e^{r_2t}$	$c_1e^{rt} + c_2te^{rt}$	$c_1e^{at}\cos(bt) + c_2e^{at}\sin(bt)$

Once we find the general solution, we can find the solution satisfying our initial conditions by plugging them in and solving for c_1, c_2. An **Initial Value Problem (IVP)** is one where the initial conditions are of the form $y(t_0) = y_0, y'(t_0) = y_0'$. You are told the value of y and y' at the same time t_0. A **Boundary Value Problem (BVP)** is one where the initial conditions are of the form $y(a) = y_a, y(b) = y_b$. You are told the value of y at two different times $t = a, b$. An IVP has a **unique** solution, a BVP may have $0, 1$ **or** ∞ solutions.

1.2 Problems

2. True False It is possible for there to be no solutions to an initial value problem.
3. True False It is possible for there to be no solutions to a boundary value problem.
4. True False It is possible for a BVP to have only 2 solutions.
5. Solve the initial value problem given by $3y'' = 15y' - 18y$ and $y(0) = 0$ and $y'(0) = 1$.
6. Solve the initial value problem $2y'' + 4y' + 2y = 0$ with $y(0) = 0, y'(0) = 1$.
7. Solve the boundary value problem of a mass on a spring given by $x'' = -4x$ and $x(0) = 0, x(\pi) = 0$.
8. Solve the boundary value problem given by $y'' = -y$ and $y(0) = 0, y(\pi) = 1$.
9. Find the second order linear ODE such that $y(t) = e^{2t}\sin(t)$ is a solution to it.
10. What is the smallest value of $\alpha > 0$ such that any solution of $y'' + \alpha y' + y = 0$ does not oscillate (does not have any terms of $\sin, \cos$).